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Fig: 1

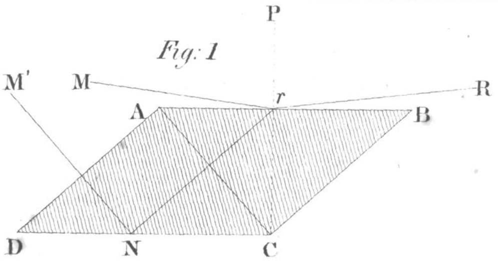


Fig: 4

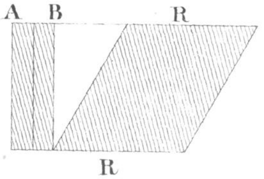


Fig: 3

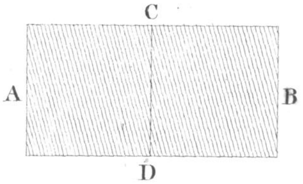


Fig: 2

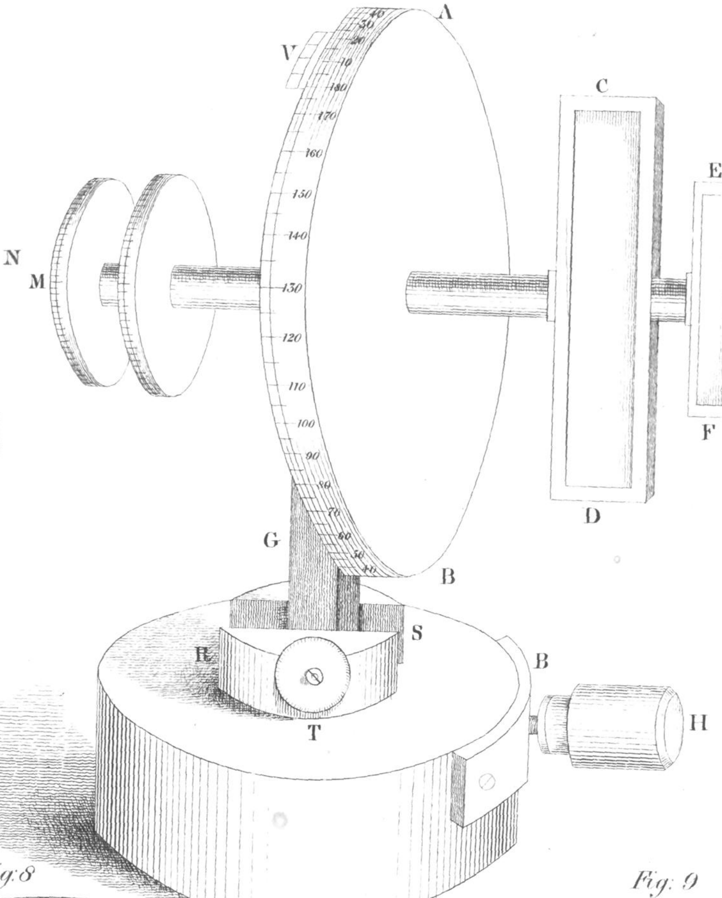


Fig: 7

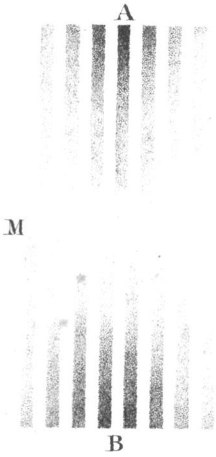


Fig: 5

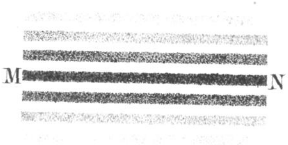


Fig: 6

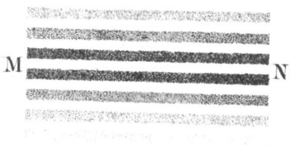


Fig: 8

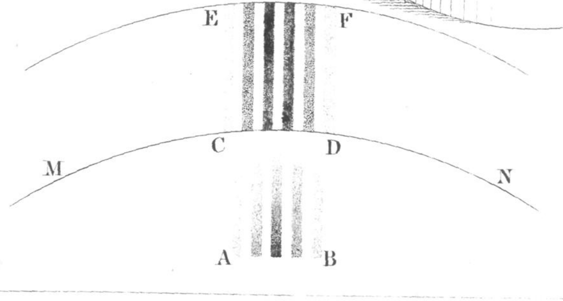
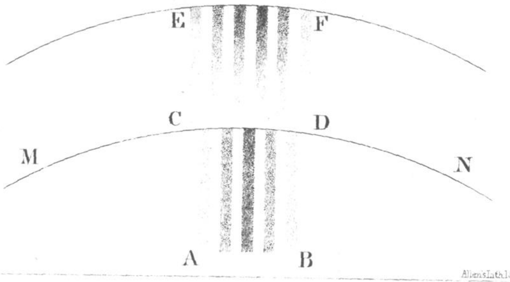


Fig: 9



XV.—*On the Compensations of Polarized Light, with the Description of a Polarimeter, for measuring Degrees of Polarization.* *By* SIR DAVID BREWSTER, K. H., D. C. L., F. R. S., M. R. I. A., and V. P. R. S. Ed.

Read November 14, 1842.

IN four papers, printed in the Philosophical Transactions for 1830, I have endeavoured to determine the general laws of the polarization of light, when reflected from or refracted by the first and second surfaces of bodies, or when suffering total or metallic reflexion. In opposition to the opinions of the most distinguished philosophers, I was led to the conclusion—that when light was reflected at any angle of incidence between 0° and 90° (excepting at the angle of complete polarization), or was refracted at these angles, it did not consist, as they maintained, of two portions, one of which was completely polarized, and the other completely unpolarized or common light; but that every portion of it had the same physical property, namely, that of having approximated more or less to the state of complete polarization. This general result, which enables us to compute all the phenomena of polarization by reflection and refraction, has, in so far as I know, never been called in question; but as the investigation was conducted on the supposition, that a pencil, composed of two pencils, polarized + 45° and — 45° to the plane of reflexion, was equivalent to a pencil of common light, it became important to have the general result confirmed by experiments made with common light itself; and though the inquiries, the results of which I am now about to explain, had not this object in view, yet it will be satisfactory to find in them a complete demonstration of my former views.*

In considering the condition of partially polarized light, it has always appeared to me probable that some method would be found of distinguishing it from a mixture of polarized and common light; and I have accordingly endeavoured at different times, though without success, to obtain such a test. While studying, however, the polarizing structure of the atmosphere, where it became desirable to ascertain the degree and kind of polarization which light reflected from different parts of it experienced, I was led to a series of experiments, which furnished me with the test of which I had been in search.

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* Philosophical Transactions, 1830, pp. 69, 133, 145, 287.

The comparative brightness of the two images in Iceland spar, directed to different parts of the sky, afforded a very imperfect indication of its state of polarization; and I had, therefore, been in the practice of employing the uniaxal or biaxal system of rings for this purpose.* Upon placing such a system between light partially polarized in one plane, and light partially polarized in an opposite plane, I found that the rings disappeared, the direct system being seen on one side of the plane of disappearance, and the complementary system on the other side.

In this experiment, the polarization of the light in one plane was compensated by the polarization of the same light in the opposite plane, and consequently both of the pencils that had undergone the two successive polarizing actions, had received the same degree of polarization in opposite planes. In virtue of these two equal and opposite polarizations, the light at the point of compensation, where the system of rings disappeared, had been restored from partially polarized to common light, and the light on each side of this point of compensation was in opposite states of partial polarization.

In order to have a more distinct idea of the nature of this experiment, let us suppose that light reflected once, at 24° of incidence, from glass, whose index of refraction is 1.525, is afterwards made to suffer one refraction at 80° by another surface of the same glass.† In this case, the partial polarization produced by reflexion is exactly compensated by the equal and opposite partial polarization produced by refraction. In like manner, a second reflexion at $83\frac{1}{2}$°, in an opposite plane, will compensate the first reflexion at 24°, or the refraction in the same plane at 80°.

Now, in these three cases of compensation, the quantity of polarized light in the three pencils is very different, as appears from the following table:

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* See my Treatise on New Philosophical Instruments, 1813, p. 349.

† The action of one refraction is obtained by using a prism of well annealed glass, as shown in the Philosophical Transactions, 1830, p. 135, fig. 2.

Angles of Incidence.	No. of Rays, out of 1000 polarized by Reflection and Refraction.
24°	10.5
80°	158
83½°	139.3

Hence, it is obvious that the compensation is not produced by equal quantities of light polarized in opposite planes; and it would be absurd to suppose that the portions of common light existing in each of the partially polarized pencils performed any part in the compensation. But even if it did, it could act only by its quantity—that is, by the relation which it bore to the polarized portion of the beam. Now, in the three cases which we have noticed, the ratio of the common to the polarized portion of the pencil is not the same, although the compensation is perfect, as the following numbers show:

	Ratio of common and polarized Light.	
Angles of Incidence.	Reflected Pencil.	Refracted Pencil.
24°	4.15 to 1	. . . . .
80°	. . . . .	2.8 to 1
83½°	2.8 to 1	. . . . .

Hence, we are forced to the conclusion, that the compensation is produced neither by an equality of oppositely polarized rays, nor by a proportional admixture of common light, but by equal and opposite physical states of the whole pencil, whether reflected or refracted.

Let us now consider what takes place at the polarizing angle, or 56° 45′, in glass. The whole of the reflected light, or 79½ rays, is here wholly polarized, and the same quantity of oppositely polarized light, viz. 79½ rays, exists in the refracted beam. Now, this refracted beam is not capable of compensating the reflected one, notwithstanding their equality in point of polarized light, and though the reflected beam is not mixed with common light; so that, upon the old hypothesis, the refracted beam can owe its deficient power of compensation only to the large quantity of common light which it contains.

But though in the compensations already mentioned the proportions of common to polarized light are different; yet, in other cases of compensation, such as the following, the proportion is pretty nearly equal; but this equality is accidental, and is not the cause by which the compensation is produced.

Angles of Incidence.	Reflected Pencil.	Light polarized.	Refracted Pencil.	Ratio of common to polarized Light.
15° 40′	43.4	4.5	. . .	1/9.6
56° 45′	79.5	79.5	920	1/11.6
87° 51′	80.9	70	. . .	1/11.5

Hence, a pencil reflected at an incidence of 15° 40′, compensates another reflected at 87° 51′, and each of them compensates a pencil refracted at the polarizing angle 56° 45′, and the ratio of the common to the polarized light is nearly the same.

In support of the same views we shall examine what takes place at other three remarkable angles of incidence.

1. At 78° 7′ where the quantity of polarized light is a maximum, or 158 rays, the power of compensation by reflexion is less than at every angle of incidence between 78° 7′, and 30° where the quantity of polarized light varies from 158 to 17 rays.

At 78° 7′ the quantity of refracted light is double that of the reflected light, and is equal to two-thirds of the incident light, and the quantity of polarized light is nearly one-fourth of the reflected, and one-half of the refracted, light. Now, at this angle the power of compensation by reflexion and refraction is nearly in the inverse ratio of the quantity of light in the reflected and refracted beams, and not as the quantities of common light, which they are supposed to contain. For the powers of compensation are as 6° 50′ to 14° 7′; the ratio of light in each beam as 666 + 333, and the proportion of common light as 508 to 175.

2. At 85° 50′ 40″, when $i - i' = 45°$, when the refracted is one-half of the reflected light, and the quantity of polarized light one-third of the refracted light, one-sixth of the reflected light, and one-ninth of the incident light, the power of compensation by refraction is nearly double of that by reflexion,* being nearly in the inverse ratio of the quantities of light in the reflected and refracted beams, and not of the quantities of common light which they contain.

At other angles of incidence beside these two, the powers of compensation have no such relations.

3. At 82° 44′, a very remarkable angle, where $\cos(i + i') = \cos.^2 (i - i')$, and where the reflected is equal to the refracted light, the compensation by reflection is equal to the compensation by refraction, and the ratio of the polarized

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* The one is 9° 44′, and the other 4° 48′.

to the common light, or to the total quantity in each beam, is the same;* but this equality is accidental, as appears from the fact already mentioned.

The remarkable phenomena produced at this angle in glass, and at the corresponding angle in all transparent bodies, where cos. $(i+i') = \cos.^2 (i-i')$ require to be more minutely stated, and lead us to the construction of what may be called the *compensating rhomb*, which is shown in Plate, Fig. 1. It consists of a well annealed rhomb of glass, or of any other uncrystallized body ABCD, having, in the case of glass, the angles BAD, BCD = 139° 25′, and ABC = 40° 35′, when the index of refraction is 1.525. If a ray of light R*r*, is incident upon AB, at an angle of 82° 44′, exactly one-half of it will be reflected in the direction *r*M, and the other half refracted in the direction *r*N, having each the same quantity of polarized light, as already stated. But the ray *r*N is again reflected at N at an angle of 40° 35′, and it will emerge from the face AD nearly perpendicularly, without suffering any perceptible refraction, in the direction NM′. If we now examine this ray M′N, we shall find it to be in the state of common light, although the incident ray *r*N contained 145 polarized rays, or nearly one-half of the pencil *r*N. In order to be satisfied of this, the compensating rhomb should be made of two equal and similar rectangular prisms, ABC, ADC, cemented to or nearly touching one another. By removing ADC, the ray *r*N emerging nearly perpendicularly from the face AC, will exhibit the state of its polarization, when it falls upon the face DC at the point N.

We have now obtained by this experiment a very singular result. If the pencil *r*N consists of 145 rays of polarized light, and 333 — 145 = 188, of common light, the effect of a single reflexion at N has been to unpolarize polarized light! and to produce no change at all upon common light! a property of a reflecting surface hitherto unheard of, and incompatible with all our present knowledge of the polarization of light. After such a conclusion, it would be an unprofitable task to adduce any further arguments; and I shall therefore only state that all the phenomena of polarization, by successive reflexions and refractions, stand in direct contradiction of the views which I have been combating.

The restoration of the pencil *r*N to common light by reflexion at N, furnishes us at once with the principle of compensation, in conformity with the laws of polarization deduced in my papers of 1830. The whole of the ray *r*N has suffered a physical change by refraction at *r*, consisting of a rotation of its planes of polarization towards a plane perpendicular to that of refraction, and the subsequent reflexion at N has exactly counteracted that rotation, by turning back the planes as many degrees towards the plane of reflexion. The reflexion at N has, therefore, brought back the ray *r*N into the same state as the original ray R*r*, that is, the ray NM′ is common light.

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* This is the only angle where this equality obtains.

In order to ascertain if this principle is general, and to determine the laws of the compensation of partially polarized light, I shall now describe the instrument by which I have ascertained the physical condition on which compensation depends, and the leading facts on which the doctrine rests. From its property of measuring degrees of polarization, I have called this instrument a *Polarimeter*. It is represented in Fig. 2, and consists of two parts, one of which is intended to produce a ray of compensation, having a physical character susceptible of numerical expression, and the other to produce polarized bands, or rectilinear isochromatic lines, the extinction of which indicates that the compensation is effected. The first part of the instrument consists of a goniometer AB, carrying on its axis MN, a frame CD containing six or seven plates of glass, about the 70th of an inch thick, such as are now used for holding microscopic objects. This frame can be taken off and replaced by a black glass reflector highly polished, and free of all oxidation on its surface, or it may be fixed permanently at EF, alongside of the frame CD.*

The second part of the *Polarimeter* is a combination of two plates of rock crystal, or any other transparent doubly refracting mineral, such as I described in 1819, in my paper *on the Properties of Amethyst.*† The object which I had in view by this combination was to exhibit the colours of polarized light in *rectilinear bands*, and this is effected in the following manner. A plate of rock crystal, AB, Fig. 3, from the fiftieth to the tenth of an inch thick, is cut so that its faces are inclined 45° to the axis of the prism, which is the axis of double refraction. When the plate has been divided into two equal parts at the line CD, the one is placed transversely above the other, and cemented to it by Canada balsam, so that the two plates act in opposition to each other upon polarized light. When this plate is fixed at the end of a Nicol's prism (or a rhomb of calcareous spar, with a circular aperture just sufficient to separate the two images), as shewn in Fig. 4, the depolarizing axis of the plate being parallel to the principal section of the rhomb, we shall observe in polarized light a beautiful system of rectilineal bands, as exhibited in Fig. 5, where MN is a deep black neutral line, with the usual coloured bands on each side of it. With light polarized oppositely, the central band MN is white, as shewn in Fig. 6, in which the tints are complementary to those in Fig. 5.

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* When much light is desired, a plate of a highly refracting substance, whose index of refraction is known, may be substituted for the glass.

† See Edinburgh Transactions, vol. ix. p. 148.

Let us now suppose it required to determine the state or degree of polarization of any luminous surface from which light is reflected, or through which it is transmitted, or of any illuminated medium from which both reflected and refracted light are transmitted to the eye of the observer.

If the light is polarized in the plane of the meridian or a vertical plane, it may be more convenient to use the glass plate at CD, and in doing this the analyser with the rock crystal is fixed between the frame CD and the eye of the observer upon a pillar, or it may be held in the hand, so that the principal section of the rhomb is in a vertical plane. The rectilineal bands will then be seen distinctly crossing the luminous surface, when CD is perpendicular to the axis of vision. But if we incline CD from 0° of incidence towards 90, by turning round the goniometer, a position will be found when the rectilinear bands are interrupted by a neutral line, as in Fig. 7, and the bands at A on one side of the neutral line will be complementary to those at B on the other side. After marking the indication of the goniometer, when this takes place, suppose 160°, turn back the goniometer till the light from the luminous surface is nearly as much inclined to the plates on the other side of 0° of incidence, and vary the angle till the bands are interrupted as before, when the observer looks at the same point of the luminous surface. Having again observed the indication of the goniometer, suppose 10°, then $160° - 10° = 150°$ will be the inclination of two rays equally inclined to the plate, and the half of this, or 75°, will be the angle of incidence upon the plates, at which the polarization of the light from the luminous surface is compensated.

If the light from the luminous surface had been polarized horizontally, it would have been most convenient to have used the rock glass, or other reflector not metallic. In doing this, the luminous surface is reflected at the same angle between the polarizing angle and 90°, and also between the same angle and 0°, the analyser and rhomb being in each case interposed between the reflector and the eye, as before, and the angle of incidence varied till the neutral line MN is opposite to or seen upon the same part of the luminous surface. If the compensation takes place about 70°, it will also take place about 40°, and these angles will afford measures of the degree of polarization necessary to produce the compensation required.

In order to make these observations at different inclinations to the horizon, the pillar which carries the graduated circle of the goniometer, and also the pillar GH, must move upon a joint, as shewn in Fig. 2. By observations such as those above described, the following angles of compensation will be obtained:

Compensations between two Reflexions, one above and one below the polarizing Angle.

Below polarizing Angle.	Above polarizing Angle.
0°	90°
5	89½
10	89
16	88
20	86
24	83
30°	81°
34	77
39	74
44	70
48	65
52	62
56¾	56¾

Compensations between one Reflexion below the polarizing Angle, and one Refraction.

Reflexion.	Refraction.
2°	10°
5	22
10	42
15	59
20	74
24	80
25	81
28	87
30	89½

Compensations between one Reflexion above the polarizing Angle, and one Refraction.

Reflexion.	Refraction.
$89\frac{1}{2}$	22°
89	42
88	59
86	74
83	80
82	87
81	$89\frac{1}{2}$

If we now compare these results with the experimental and calculated ones given in my papers of 1830,* we shall find that one reflexion will compensate another reflexion, or one refraction, when the inclinations of the planes of polarization produced by the two reflexions are equal and opposite, or when the inclination produced by one reflexion is the complement of the inclination produced by one refraction; or more generally, in both cases, when the rotations produced in the plane of polarization are equal and opposite. Hence, it follows that the compensations of polarized light are produced by equal and opposite rotations of the planes of polarization.

Now, the inclination ϕ of the plane of polarization by reflexion at any angle of incidence i, is

$$\tan\phi = \tan.\,x\,\frac{\cos(i+i')}{\cos(i-i')},$$

and the inclination ϕ' for refracted light, is $\cot\phi' = \cot x\,.\,\cos(i-i')$. In the case of reflected light, the angles of incidence which compensate each other are those where ϕ has equal values; and in the case of reflected and refracted light, the one compensates the other, when $\phi + \phi' = 90°$, or $\tan\phi + \cot\phi' = 1$, or when

$$\tan x\,\frac{\cos(i+i')}{\cos(i-i')} + \cot x\,.\,\cos(i-i') = 1.$$

Now, though we shall find that at the angles of compensation in the preceding table, the values of $+\phi$ and $-\phi$ in the case of reflexion, and of $\pm\phi$ and $90^\circ - \phi'$ in the case of a reflexion, and a refraction, are nearly equal; yet it requires to be proved, that when the planes of polarization are inclined at an angle $\pm x$, to the plane of incidence, greater or less than 45°, another reflexion at another angle, which would give $\pm\phi$, or $90^\circ - \phi'$, of the same value, will restore the planes to their original inclination.

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* Philosophical Transactions, 1830, pp. 74, 75, 78; 136, 138, 139, and 143.

When $x = 45^\circ$, and when one reflexion has turned the planes of a ray polarized 45° into 37° 21′, or given the planes a rotation of $45^\circ - 37^\circ\,21' = 7^\circ\,39'$, the action of a refracting surface which produces the same rotation, or $52^\circ\,39' - 45^\circ = 7^\circ\,39'$ will bring the planes back to 45°, or restore the partially polarized light to common light. Call $x = 37^\circ\,21'$, then in order that the refraction may restore the ray to 45° we must have $\phi' = 45^\circ$ or $\cot\phi' = \cot x \cos(i - i') = 1$. Now, $\cot\phi' = \cot x \cos(i - i')$, and when $x = 45^\circ$ and $\phi' = 52^\circ\,39'$, $\cot\phi' = \cos(i - i')$. But $x = 37^\circ\,21' = 90^\circ - \phi$, hence $\frac{1}{\cot x} = \cot\phi'$, and $\frac{1}{\cot x} = \cos(i - i')$, consequently $\cot x \cot(i - i') = 1$. In like manner ϕ' will be restored to 45° by a reflexion which gives ϕ such, that $\phi + \phi' = 90^\circ$, or $\tan\phi = \cot\phi'$. That is when $x = 45^\circ$, and $\phi = 37^\circ\,21'$,

$$\tan\phi = \tan x\,\frac{\cos(i + i')}{\cos(i - i')} = 1.$$ The general formula

$$\tan\phi = \tan x\,\frac{\cos(i + i')}{\cos(i - i')}$$ becomes, when $x = 45^\circ$,

$$\tan\phi = \frac{\cos(i + i')}{\cos(i - i')},$$

But when $x = 52^\circ\,39' = 90^\circ - \phi$, we have

$$\frac{1}{\tan x} = \tan\phi, \text{ and}$$

$$\frac{1}{\tan x} = \frac{\cos(i + i')}{\cos(i - i')}.$$ Consequently,

$$\tan x\,\frac{\cos(i + i)}{\cos(i - i)} = 1.$$

Having thus determined that light polarized in a plane whose inclination to the plane of reflexion is $+\phi$, will be compensated by oppositely polarized light, whose inclination is $-\phi$, if both the lights are reflected, or by refracted light whose inclination is $90^\circ - \phi$ or ϕ', we must next endeavour to discover at what angle of incidence the polarized light submitted to the polarimeter, has suffered reflexion or refraction, when we have the angle of incidence and the inclination of the plane of polarization, by which we have effected the compensation.

Let us first take the case when light partially polarized by reflexion is compensated by the polarization produced by refraction through one surface, at an incidence i of 80°. The index of refraction being 1.525, we shall have when

$$x = 45^\circ, \ \cot\phi' = \cos(i - i'), \text{ and } \phi' = 52^\circ\,33'.$$

Now, the plane of the light polarized by reflexion must be inclined $90^\circ - \phi'$, or $37^\circ\,27'$; we must, therefore, find the angles of incidence above and below the polarizing angle, or the two values of i corresponding to this value of ϕ, namely, $37^\circ\,27'$, at one or other of which the original light must have been reflected. These values will be obtained from the expressions

$$\tan\phi = \frac{\cos(i + i')}{\cos(i - i')}, \text{ and } \sin i' = \frac{\sin i}{m}.$$

When $i + i'$ is less than 90°, or when the angle of incidence is less than the polarizing angle, $\tan\phi$ is *positive*, and we have

$$\sin i = \sqrt{\frac{(m^2+1)(1-\tan\phi)^2}{8\tan\phi}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{4\tan\phi}{(1-\tan\phi)^2}}\right\}}\,.$$

When $i + i'$ is greater than 90°, and $\tan\phi$ *negative*, the formula becomes

$$\sin i = \sqrt{\frac{(m^2+1)(1+\tan\phi)^2}{-8\tan\phi}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{-4\tan\phi}{(1+\tan\phi)^2}}\right\}}\,.$$

From these formulæ, whem $m = 1.525$ and $\phi = 37^\circ\,27'$, we obtain $i = 24^\circ\,50'$, and $83^\circ\,30'$.

When the compensation of *refracted* light is effected by one *reflexion*, either above or below the polarizing angle, for example, at 15° 40', and 87° 51', we shall have

$$\tan\phi = \frac{\cos(i+i')}{\cos(i-i')} = 42^\circ\ 31'.$$

But in the refracted light thus compensated, we must have $\phi' = 90^\circ - 42^\circ\ 31 = 47^\circ\ 29'$, and, therefore, we must determine the angle of incidence i, at which the original light suffered refraction. The expressions from which we obtain i are $\cot\phi' = \cos(i-i')$, and $\sin i' = \frac{\sin i}{m}$, which give

$$\sin i = \frac{m}{\tan\phi'}\sqrt{\frac{\tan^2\phi' - 1}{m^2+1-\frac{2m}{\tan\phi'}}},$$

from which we obtain, when $\phi' = 47^\circ\ 29'$, $i = 56^\circ\ 45'$, the maximum polarizing angle.

Hitherto we have supposed the compensation to be produced by one refraction, or by one reflexion; but it may be effected by several. In the case of reflected light this is not necessary, because we have all degrees of polarization by reflexion, from 0° of incidence to the polarizing angle, and from this again to 90° of incidence.

When the compensation, however, is made by *successive reflexions* at the same angle of incidence, or when light which is compensated has been so reflected, we may find the angle of incidence i, when n is the number of reflexions, by means of the formulæ

$$\tan\phi = \frac{\cos^n(i+i')}{\cos^n(i-i')},\quad \sin i' = \frac{\sin i}{m},\quad \text{and}\quad \sqrt[n]{\tan\phi} = \frac{\cos(i+i')}{\cos(i-i')},^*$$

which give

$$\sin i = \sqrt{\frac{(m^2+1)(1-\sqrt[n]{\tan\phi})^2}{8\sqrt[n]{\tan\phi}}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{4\sqrt[n]{\tan\phi}}{(1+\sqrt[n]{\tan\phi})^2}}\right\}},$$

when $i+i'$ is less than the polarizing angle, and $\tan\phi$ *positive*. But when $i+i$ is greater than 90°, and $\tan\phi$ *negative*, we have

$$\sin i = \sqrt{\frac{(m^2+1)(1+\sqrt[n]{\tan\phi})^2}{-8\sqrt[n]{\tan\phi}}\left\{-1 \pm \sqrt{1+\left(\frac{2m}{m^2+1}\right)^2 \times \frac{-4\sqrt[n]{\tan\phi}}{(1+\tan\phi)^2}}\right\}}.$$

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* See Phil. Trans. 1830, p. 80, 81.

In the case of light polarized by *refraction*, the action of several surfaces may and must often be necessary to produce compensation, and in this case, or when the light compensated is polarized by *successive refractions*, we may find the angle of incidence by means of the formulæ

$$\cot \phi = \cos^n (i - i'), \text{ and } \sin i' = \frac{\sin i}{m}.$$

And since $\sqrt[n]{\cot \phi} = \cos (i - i')$, we have for n refractions,†

$$\sin i = \frac{m}{\sqrt[n]{\tan \phi}} \sqrt{\frac{\sqrt[n-1]{\tan \phi - 1}}{m^2 + 1 - \frac{2m}{\sqrt[n]{\tan \phi}}}}.$$

When the light has passed through a prism whose angle is ψ, then if the angle of the prism is equal to the angle of refraction, or $\psi = i'$, or $\sin \psi = \frac{\sin i}{m}$, the incidence i will be found from the formula for *one* refraction, because the ray will emerge perpendicularly from the second surface of the prism, and suffer no change in its planes of polarization.

If the angle of the prism is double the angle of refraction, or $\psi = 2i'$, and the deviation $i - i'$ a *minimum*, the incidence i will be found from the formula when n, the number of refractions, is *two ;* the refraction, and consequently, the polarization at each surface being equal, and, therefore, the same, as for a plate when $\psi = 0$.

Having thus determined the laws of the compensation of polarized light, I shall conclude this paper by pointing out a few of their numerous applications.

1. The first and most important result of this inquiry is, that it affords a new and independent demonstration of the laws of the polarization of light by reflexion and refraction, given in my papers of 1830. As this result has been already referred to, I shall merely mention the following general proposition.

When a ray of common light is incident at any angle upon the polished surface of a transparent body, the whole of the reflected pencil suffers a physical change, bringing it more or less into a state of complete polarization ; in virtue of which change, its planes of polarization are more or less turned into the plane

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[*] See Phil. Trans. 1830, p. 137.

of reflexion, while the whole of the refracted pencil has suffered a similar, but opposite change, in virtue of which, its planes of polarization are turned more or less into a plane perpendicular to the plane of reflexion.

2. As the light of the sky and the clouds is more or less polarized, the employment of the light which they reflect may, in delicate experiments, be a serious source of error, if we are not aware of its properties. By the principle of compensation, however, we may convert this partially polarized light into common light, and thus make experiments with as great accuracy in the day-time, as we can do with the direct light of a flame. If the light from a particular part of the sky is admitted into a dark room, or otherwise employed, we have only to compensate its polarization either by reflexion or refraction, and employ, as *unpolarized* or *common light,* that part of the light which corresponds with the neutral line.

3. The laws of the compensation of polarized light enable us to investigate the polarizing structure of the atmosphere, and to ascertain the nature and extent of the two opposite polarizing influences, which I have found to exist in it, and by the compensation of which the neutral points are produced. But, as I shall soon submit to the Society the results of my observations on this subject, I shall not add any thing further at present.

4. In every case where reflected or refracted light reaches the eye of the observer, whether it comes from bodies near us, or from the primary or secondary planets of our system, the doctrine of compensation enables us to obtain important information respecting the phenomena presented by light thus polarized. The nature of the reflecting or refracting surface, the angles of reflexion or refraction, and the nature of the source of illumination, may, in certain cases, be approximately ascertained.

5. When the light of the sun, or any self-luminous body, is reflected from the surface of standing water, such as the sea or a lake, it is polarized according to laws which are well known ; but when the partially polarized light of the sky (light polarizes in every possible plane, passing through the sun and the observer) is reflected, a variety of curious compensations take place, which, when the position of the observer is fixed, vary with the season of the year, and the hour of the day. In some cases, there is a perfect compensation, the partially polarized light of the sky being restored to common light by the reflection of the

water. In other cases the light of the sky has its polarization increased by reflexion from the water in the same plane in which it was itself polarized; and in other cases, the compensation is effected only in particular planes. At sunset, for example, the light reflected from the sea at a great obliquity in two vertical planes inclined 45° to a vertical plane passing through the sun and the observer, is compensated in these two planes, or the plane of its polarization is inclined about 45° to the reflecting surface. The same observations apply to the light of the two rainbows when reflected from the surface of water.

6. When the light of the sky, or of the rainbow, is reflected from surfaces not horizontal, such as the roofs of houses, sheets of falling water, or surfaces of smoke and vapour, the compensations are more varied, and a perfect neutralization of the light by the second reflexion is more frequently obtained.

7. When the compensating rhomb, whose properties I have already described, is made of glass not highly polished, light that has suffered total reflexion is seen through the face AD, Fig. 1. As the faces AB, CD, are parallel, none of the light regularly refracted by the face AB can suffer total reflexion from CD. Upon examining this curious and unexpected phenomenon, I found that it was owing to light radiated, or scattered from the surface AB, which falling upon CD at angles greater than that of total reflexion, whose sine is $\frac{1}{m}$, necessarily suffered total reflexion. That this was the cause of the phenomenon, I proved by covering the surface AB with a film of dried milk, which radiated light from every part of its surface, and produced a beautiful zone of totally reflected light, increasing in brightness as the incidence upon AB became more oblique. In examining this totally reflected light, I was greatly surprised to find, that it was partially polarized, and exhibited an interesting example of compensation.

Let MN, Fig. 8, be the luminous zone of totally reflected light with its blue border. At the polarizing angle of the second surface of the rhomb, the polarization is incomplete; but at angles between that angle and 83°, the polarization gradually diminishes, and at 83° it becomes common light, the rectilineal fringes AB produced by the rock crystal passing into neutral light at CD, close to the boundary MN of total reflexion. From 83° to 90°, which corresponds to a very narrow space at CD, the light still appears compensated, though it is slightly polarized, in a plane perpendicular to that of reflexion.

At 83°, when this takes place, the totally reflected light MN is polarized, as shown at EF, Fig. 9, in a plane at right angles to that of reflexion. But as the angle of incidence diminishes, the polarization diminishes, till at an angle of 68° it becomes common light, the polarization produced by total reflexion at the second surface exactly compensating, as at CD, that produced by refraction at the first.

At angles less than 68°, the totally reflected light is partially polarized in the plane of reflexion, the polarization increasing till the scattered light disappears.

The polarization of the light that afterwards suffers total reflexion, is produced by its refraction at the first surface AB, Fig. 1 of the rhomb, and the phenomena above described arise from the opposite action of the reflecting surface CD; at one angle producing an inferior degree of polarization, at another compensating it, and at another overbalancing it.

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